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SHORT TRICKS

Mathematics

for JEE (Main & Adv)

Sunil Gupta
Career Point Editorial

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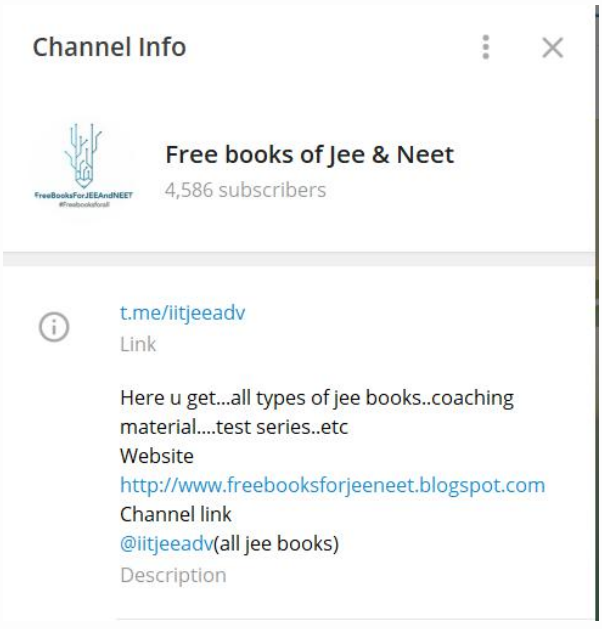
Mathematics
for JEE (Main & Advanced)



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6

Determinant

KEY CONCEPTS

1. Definition

An expression expressed in equal number of rows and column and put between two vertical lines is named as determinant.

e.g. $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$, $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ are determinant of second and third order.

2. Expansion of Determinant

(i) Expansion of two order : $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1b_2 - a_2b_1$

(ii) Expansion of three order : (with respect to first row)

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$$

$$= a_1(b_2c_3 - b_3c_2) - b_1(a_2c_3 - a_3c_2) + c_1(a_2b_3 - b_2a_3)$$

3. Minor and Cofactor

(i) Minor :

If $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ then

Minor of a_{11} is $M_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$,

Similarly minor of a_{12} is $M_{12} = \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$

(ii) Cofactor :

The cofactor of an element a_{ij} is denoted by F_{ij} & is equal to

$(-1)^{i+j} M_{ij}$ where M is a Minor of element a_{ij}.

If $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

then $F_{11} = (-1)^{1+1} M_{11} = M_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$

and $F_{12} = (-1)^{1+2} M_{12} = -M_{12} = -\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$

4. Properties of minor and cofactor

(i) The sum of products of the element of any row with their corresponding cofactor is equal to the value of determinant i.e.
$\Delta = a_{11}F_{11} + a_{12}F_{12} + a_{13}F_{13}$

(ii) The sum of the product of element of any row with corresponding cofactor of another row is equal to zero i.e. $a_{11}F_{21} + a_{12}F_{22} + a_{13}F_{23} = 0$

(iii) If order of a determinant (Δ) is 'n' then the value of the determinant formed by replacing every element by its cofactor is Δ^{n-1}.

5. Properties of Determinants

(i) The value of the determinant remains unaltered, if two rows and columns are interchanged. This is denoted by 'and is also called transpose.

e.g. $A = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = A'$

(ii) If any two rows (or columns of a determinant be interchanged, the determinant is unaltered in numerical value, but is changed in sign only.

e.g. $A = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ and $B = \begin{vmatrix} a_2 & b_2 & c_2 \\ a_1 & b_1 & c_1 \\ a_3 & b_3 & c_3 \end{vmatrix}$ then $B = -A$

(iii) If all the element of a row in Δ are zero or two rows (columns) are identical (or proportional), then the value of Δ is zero.

(iv) If all the elements of any row (or column) be multiplied by the same number, then the determinant is multiplied by that number.

e.g. $A = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ and $B = \begin{vmatrix} ka_1 & kb_1 & kc_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ then $B = kA$

(v) The value of a determinant is not altered by adding the elements of any row (or column) to the same multiple of the corresponding elements of any other row (or column).

e.g. let $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 + mb_1 - c_1 & b_1 & c_1 \\ a_2 + mb_2 - c_2 & b_2 & c_2 \\ a_3 + mb_3 - c_3 & b_3 & c_3 \end{vmatrix}$

(vi) If Δ becomes zero on putting $n = \alpha$ then we say that $(n - \alpha)$ is a factor of Δ.
If each element of any row (column) can be expressed as a sum of two terms, then the determinant can be expressed as the sum of two determinants.

$$\text{e.g. } \begin{vmatrix} a_1+x & b_1+y & c_1+z \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} x & y & z \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

(vii) $$\begin{vmatrix} a_1 & a_2 & a_3 \\ 0 & b_2 & b_3 \\ 0 & 0 & c_3 \end{vmatrix} = a_1b_2c_3 = \begin{vmatrix} a_1 & 0 & 0 \\ b_1 & b_2 & 0 \\ c_1 & c_2 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & 0 & c_3 \end{vmatrix}$$

(viii) If all the elements below leading diagonal or above leading diagonal or except leading elements are zero then the value of the determinant equal to multiplied of all diagonal element.

$$\text{e.g. } \begin{vmatrix} a_1 & b_1 & c_1 \\ 0 & b_2 & c_2 \\ 0 & 0 & c_3 \end{vmatrix} = a_1b_2c_3$$

6. Multiplication of Two Determinants

Multiplication of two determinants is possible if they are of same order.
Multiplication of two third order Determinants is defined as follows

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \times \begin{vmatrix} \ell_1 & m_1 & n_1 \\ \ell_2 & m_2 & n_2 \\ \ell_3 & m_3 & n_3 \end{vmatrix}$$

$$= \begin{vmatrix} a_1\ell_1 + b_1\ell_2 + c_1\ell_3 & a_1m_1 + b_1m_2 + c_1m_3 & a_1n_1 + b_1n_2 + c_1n_3 \\ a_2\ell_1 + b_2\ell_2 + c_2\ell_3 & a_2m_1 + b_2m_2 + c_2m_3 & a_2n_1 + b_2n_2 + c_2n_3 \\ a_3\ell_1 + b_3\ell_2 + c_3\ell_3 & a_3m_1 + b_3m_2 + c_3m_3 & a_3n_1 + b_3n_2 + c_3n_3 \end{vmatrix}$$

Note :

- In above case the order of Determinant is same, if the order is different then for their multiplication first of all they should be expressed in the same order.

7. Symmetric Determinant

If the elements of a determinant are such that $a_{ij} = a_{ji}$, then the determinant is said to be a symmetric determinant e.g. $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$

8. Skew Symmetric Determinant

If $a_{ij} = -a_{ji}$ then the determinant is said to be a skew symmetric determinant.

$$\text{e.g. } \begin{vmatrix} 0 & b & -c \\ -b & 0 & a \\ c & -a & 0 \end{vmatrix} = 0$$

Note :

- Every diagonal element of a skew symmetric determinant is always zero.
- The value of a skew symmetric determinant of even order is always a perfect square and that of odd order is always zero.

9. Differentiation of a Determinant

There are two way's of differentiating a determinant

(i) $$\frac{d}{dx}\begin{vmatrix} R_1 \\ R_2 \\ R_3 \end{vmatrix} = \begin{vmatrix} R'_1 \\ R_2 \\ R_3 \end{vmatrix} + \begin{vmatrix} R_1 \\ R'_2 \\ R_3 \end{vmatrix} + \begin{vmatrix} R_1 \\ R_2 \\ R'_3 \end{vmatrix}$$

R_1, R_2, R_3 represent row's of determinant

(ii) $$\frac{d}{dx}|C_1\ C_2\ C_3| = |C'_1\ C_2\ C_3| + |C_1\ C'_2\ C_3| + |C_1\ C_2\ C'_3|$$

where C_1, C_2, C_3 represents column of determinant.

10. Integration of Determinant

If Δ any single row or column contain elements which are functions of x, then integration can be done in determinant format only as following.

If $\Delta = \begin{vmatrix} f(x) & g(x) & h(x) \\ a & b & c \\ p & q & r \end{vmatrix}$ where a, b, c, p, q, r all are constants, then

$$\int_a^b \Delta dx = \begin{vmatrix} \int_a^b f(x)dx & \int_a^b g(x)dx & \int_a^b h(x)dx \\ a & b & c \\ p & q & r \end{vmatrix}$$

If elements are not like above it is advisable to simplify determinants then integrate it.

11. Crammer's Rule (System of equation)

(i) System of equation involving two variable :

$a_1x + b_1y + c_1 = 0$
$a_2x + b_2y + c_2 = 0$

- **Consistent** (System of equation has solution)
  - unique solution: $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ or $a_1b_2 - a_2b_1 \neq 0$ (Equations represents intersecting lines)
  - Infinite solution: $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ (Equations represents coincident lines)
- **Inconsistent** (System of equation has no solution): $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ (Equation represents parallel disjoint lines)

If $\Delta_1 = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}$, $\Delta_2 = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$, $\Delta = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$ then $x = \frac{\Delta_1}{\Delta}$, $y = \frac{\Delta_2}{\Delta}$

(ii) System of equations involving three variables :

$a_1x + b_1y + c_1z = d_1$
$a_2x + b_2y + c_2z = d_2$
$a_3x + b_3y + c_3z = d_3$

To solve this system we first define following determinants

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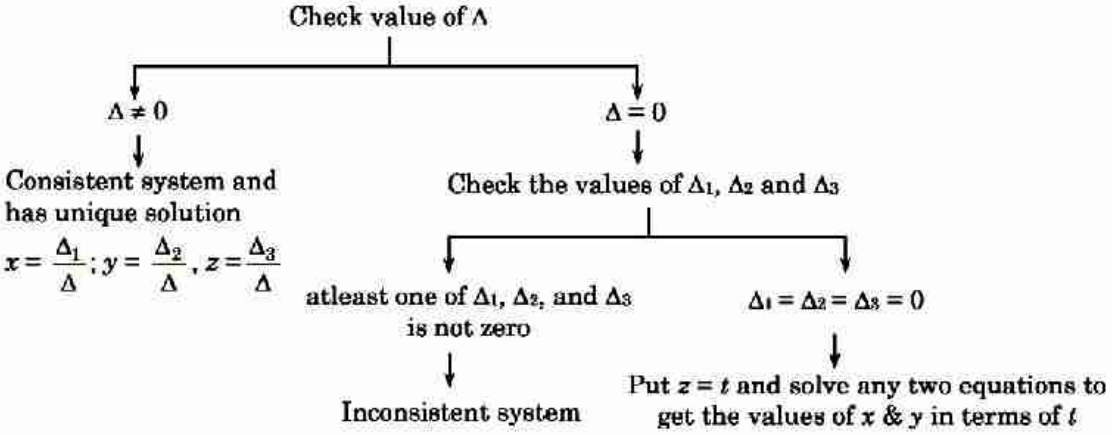
$$\Delta=\begin{vmatrix}a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\\ a_3 & b_3 & c_3\end{vmatrix}, \Delta_1=\begin{vmatrix}d_1 & b_1 & c_1\\ d_2 & b_2 & c_2\\ d_3 & b_3 & c_3\end{vmatrix}, \Delta_2=\begin{vmatrix}a_1 & d_1 & c_1\\ a_2 & d_2 & c_2\\ a_3 & d_3 & c_3\end{vmatrix}, \Delta_3=\begin{vmatrix}a_1 & b_1 & d_1\\ a_2 & b_2 & d_2\\ a_3 & b_3 & d_3\end{vmatrix}$$

then $x=\frac{\Delta_1}{\Delta}, y=\frac{\Delta_2}{\Delta}, z=\frac{\Delta_3}{\Delta}$

Now following algorithm is followed to solve the system.


Check value of Δ
Δ ≠ 0
Δ = 0
Consistent system and has unique solution
x = Δ1/Δ ; y = Δ2/Δ , z = Δ3/Δ
Check the values of Δ1, Δ2 and Δ3
atleast one of Δ1, Δ2, and Δ3 is not zero
Δ1 = Δ2 = Δ3 = 0
Inconsistent system
Put z = t and solve any two equations to get the values of x & y in terms of t

Note :

- If $\Delta = 0$ and $\Delta_1 = \Delta_2 = \Delta_3 = 0$, then system of equation may or may not be consistent :

(i) If values of x, y and z in terms of t satisfy third equation then system is said to be consistent and will have infinite solutions.

(ii) If the values of x, y, z doesn't satisfy third equation, then system is said to be inconsistent and will have no solution.

- **Trivial solution :** In the solution set of system of equation if all the variables assumes zero, then such a solution set is called trivial solution otherwise the solution are called non-trivial solution.
- If $d_1 = d_2 = d_3 = 0$ then system of linear equation are known as system of Homogeneous linear equation which always posses atleast one solution (0, 0, 0)
- If system of homogeneous linear equation posses non-zero/non-trivial solution then D = 0. In such case given system has infinite solutions.

12. Application of Determinants in Geometry

(i) The lines : $a_1x + b_1y + c_1 = 0$ .....(1)

$a_2x + b_2y + c_2 = 0$ ..... (2)

$a_3x + b_3y + c_3 = 0$ .....(3)

are concurrent if, $\begin{vmatrix}a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\\ a_3 & b_3 & c_3\end{vmatrix} = 0$,

(ii) $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a pair of straight lines if :

$$abc + 2fgh - af^2 - bg^2 - ch^2 = 0 = \begin{vmatrix}a & h & g\\ h & b & f\\ g & f & c\end{vmatrix}$$

(iii) Area of a triangle whose vertices are (x_r, y_r); $r = 1, 2, 3$ is $\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

If $\Delta = 0$ then the three points are collinear.

(iv) Equation of a straight line passing through (x_1, y_1) & (x_2, y_2) is $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$

13. Important Determinants

(i) $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ bc & ac & ab \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

(ii) $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$

(iii) $\begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca)$

(iv) $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^4 & b^4 & c^4 \end{vmatrix} = (a-b)(b-c)(c-a)(a^2+b^2+c^2-ab-bc-ca)$

(v) $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

(vi) $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ bc & ca & ab \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca)$

(vii) $\begin{vmatrix} a & bc & abc \\ b & ca & abc \\ c & ab & abc \end{vmatrix} = \begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix} = abc(a-b)(b-c)(c-a)$

(viii) $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = -a^3 - b^3 - c^3 + 3abc$

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SHORT TRICKS

Trick -1 Method of substitution

Q.1 If $\Delta_1 = \begin{vmatrix} x & b & b \\ a & x & b \\ a & a & x \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} x & b \\ a & x \end{vmatrix}$ then

(A) $\Delta_1 = 3\Delta_2^2$ (B) $\frac{d}{dx}(\Delta_1) = 3\Delta_2^2$ (C) $\frac{d}{dx}(\Delta_1) = 3\Delta_2$ (D) $\Delta_2 = 3\Delta_1^2$

Sol. **[C]**

Proper Method

Since we know that if $\Delta = f(x) = \begin{vmatrix} R_1 \\ R_2 \\ R_3 \end{vmatrix}$, then

$$\frac{d}{dx}(\Delta) = \begin{vmatrix} \frac{d}{dx}(R_1) \\ R_2 \\ R_3 \end{vmatrix} + \begin{vmatrix} R_1 \\ \frac{d}{dx}(R_2) \\ R_3 \end{vmatrix} + \begin{vmatrix} R_1 \\ R_2 \\ \frac{d}{dx}(R_3) \end{vmatrix}$$

$$\therefore \frac{d}{dx}(\Delta_1) = \begin{vmatrix} \frac{d}{dx}(x) & \frac{d}{dx}(b) & \frac{d}{dx}(b) \\ a & x & b \\ a & a & x \end{vmatrix}$$

$$+ \begin{vmatrix} x & b & b \\ \frac{d}{dx}(a) & \frac{d}{dx}(x) & \frac{d}{dx}(b) \\ a & a & x \end{vmatrix} + \begin{vmatrix} x & b & b \\ a & x & b \\ \frac{d}{dx}(a) & \frac{d}{dx}(a) & \frac{d}{dx}(x) \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 0 & 0 \\ a & x & b \\ a & a & x \end{vmatrix} + \begin{vmatrix} x & b & b \\ 0 & 1 & 0 \\ a & a & x \end{vmatrix} + \begin{vmatrix} x & b & b \\ a & x & b \\ 0 & 0 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} x & b \\ a & x \end{vmatrix} + \begin{vmatrix} x & b \\ a & x \end{vmatrix} + \begin{vmatrix} x & b \\ a & x \end{vmatrix} = 3 \begin{vmatrix} x & b \\ a & x \end{vmatrix} = 3\Delta_2.$$

Short Trick

Put $a = 1$ and $b = 0$

$\Delta_1 = x^3$ and $\Delta_2 = x^2$

$\frac{d}{dx}(\Delta_1) = 3x^2 = 3\Delta_2$

Q.2 If $\begin{vmatrix} x^2+x & x+1 & x-2 \\ 2x^2+3x-1 & 3x & 3x-3 \\ x^2+2x+3 & 2x-1 & 2x-1 \end{vmatrix} = Px - 12$ then-

(A) $P = 24$ (B) $P = -24$ (C) $P = 0$ (D) $P = 12$

Sol. **[A]**

Proper Method

Applying - $R_2 \to R_2 - (R_1 + R_3)$

Determinant $\begin{vmatrix} x^2+x & x+1 & x-2 \\ -4 & 0 & 0 \\ x^2+2x+3 & 2x-1 & 2x-1 \end{vmatrix}$

Applying $= R_1 \to R_1 + \frac{1}{4}x^2R_2$

and $R_3 \to R_3 + \frac{1}{4}x^2R_2$

Determinant $= \begin{vmatrix} x & x+1 & x-2 \\ -4 & 0 & 0 \\ 2x+3 & 2x-1 & 2x-1 \end{vmatrix}$

Applying $= R_3 \to R_3 - 2R_1$

Determinant $= \begin{vmatrix} x+0 & x+1 & x-2 \\ -4 & 0 & 0 \\ 3 & -3 & 3 \end{vmatrix}$

$= \begin{vmatrix} x & x & x \\ -4 & 0 & 0 \\ 3 & -3 & 3 \end{vmatrix} + \begin{vmatrix} 0 & 1 & -2 \\ -4 & 0 & 0 \\ 3 & -3 & 3 \end{vmatrix}$

$= x\begin{vmatrix} 1 & 1 & 1 \\ -4 & 0 & 0 \\ 3 & -3 & 3 \end{vmatrix} + \begin{vmatrix} 0 & 1 & -2 \\ -4 & 0 & 0 \\ 3 & -3 & 3 \end{vmatrix}$

$= (24x - 12) \Rightarrow P = 24.$

Short Trick

Put $x = 1$

$\begin{vmatrix} 2 & 2 & -1 \\ 4 & 3 & 0 \\ 6 & 1 & 1 \end{vmatrix} = P - 12$

$\Rightarrow 2(3) - 2(4) - 1\,(4 - 18) = P - 12$

$\Rightarrow P = 24$

Q.3 If a, b, c, are p^{th}, q^{th} and r^{th} terms respectively of a H.P., then value of the determinant $\begin{vmatrix} bc & ca & ab \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix}$ is -

(A) p + q + r (B) 0 (C) 1/p + 1/q + 1/r (D) pqr

Sol. **[B]**

Proper Method

Let A and d be the first term and common difference of the corresponding A.P., then

$\frac{1}{a} = A + (p-1)d$

$\frac{1}{b} = A + (q-1)d$

$\frac{1}{c} = A + (r-1)d$

Now det. $= abc \begin{vmatrix} 1/a & 1/b & 1/c \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix} = abc$

Short Trick

Let p = 1, q = 2, r = 3 then

a = 2, b = 3, c = 6

$\begin{vmatrix} 18 & 12 & 6 \\ 1 & 2 & 3 \\ 1 & 1 & 1 \end{vmatrix} = 18(-1) - 12(-2) + 6(-1) = 0$

$$\begin{vmatrix} A+(p-1)d & A+(q-1)d & A+(r-1)d \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix}$$

$$= abc \begin{vmatrix} A & A & A \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix} + abcd \begin{vmatrix} p-1 & q-1 & r-1 \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix}$$

$= 0 + 0 = 0$

Q.4 If $\Delta_r = \begin{vmatrix} r-1 & n & 6 \\ (r-1)^2 & 2n^2 & 4n-2 \\ (r-1)^3 & 3n^2 & 3n^2-3n \end{vmatrix}$, then $\sum_{r=1}^{n} \Delta_r$ equals -

(A) 1 (B) -1 (C) 0 (D) n

Sol. **[C]**

Proper Method

$$\because \sum_{r=1}^{n}(r-1) = 1 + 2 + \ldots + (n-1) = \frac{n(n-1)}{2}$$

$$\sum_{r=1}^{n}(r-1)^2 = 1^2 + 2^2 + \ldots + (n-1)^2 = \frac{n(n-1)(2n-1)}{6}$$

$$\sum_{r=1}^{n}(r-1)^3 = 1^3 + 2^3 + \ldots + (n-1)^3 = \frac{n^2(n-1)^2}{4}$$

$$\therefore \sum_{r=1}^{n}\Delta_r = \begin{vmatrix} \frac{n(n-1)}{2} & n & 6 \\ \frac{1}{6}n(n-1)(2n-1) & 2n^2 & 2(2n-1) \\ \frac{1}{4}n^2(n-1)^2 & 3n^3 & 3n(n-1) \end{vmatrix}$$

$$= \frac{n(n-1)}{12} \begin{vmatrix} 6 & n & 6 \\ 2(2n-1) & 2n^2 & 2(2n-1) \\ 3n(n-1) & 3n^3 & 3n(n-1) \end{vmatrix} = 0$$

Short Trick

Put $r = 1$, $n = 1$

$$\Delta_1 = \begin{vmatrix} 0 & 1 & 6 \\ 0 & 2 & 0 \\ 0 & 3 & 0 \end{vmatrix} = 0$$

Q.5 If $D_r = \begin{vmatrix} r & x & n(n+1)/2 \\ 2r-1 & y & n^2 \\ 3r-2 & z & n(3n-1)/2 \end{vmatrix}$, then $\sum_{r=1}^{n} D_r$ is equal to -

(A) $\frac{1}{6}n(n+1)(2n+1)$ (B) $\frac{1}{4}n^2(n+1)^2$ (C) 0 (D) $\frac{n(n+1)}{2}$

Sol. **[C]**

Proper Method

$$D_r = \begin{vmatrix} r & x & n(n+1)/2 \\ 2r-1 & y & n^2 \\ 3r-2 & z & n(3n-1)/2 \end{vmatrix}$$

$$\text{then } \Sigma D_r = \begin{vmatrix} \Sigma r & x & n(n+1)/2 \\ \Sigma(2r-1) & y & n^2 \\ \Sigma(3r-2) & z & n(3n-1)/2 \end{vmatrix}$$

Short Trick

$r = 1$, $n = 1$

$$D_r = \begin{vmatrix} 1 & x & 1 \\ 1 & y & 1 \\ 1 & z & 1 \end{vmatrix} = 0$$

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$\therefore \Sigma r = n(n+1)/2$

$\Sigma(2r-1) = 2\Sigma r - \Sigma 1 = \frac{2n(n+1)}{2} - n = n^2$

& $\Sigma(3r-2) = 3\Sigma r - 2\Sigma 1 = \frac{3n(n+1)}{2} - 2n$

$= \frac{n(3n-1)}{2}$

$\therefore C_1$ & C_3 are identical so, $\Sigma D_r = 0$

Q.6 If $\begin{vmatrix} a^2 & b^2 & c^2 \\ (a+1)^2 & (b+1)^2 & (c+1)^2 \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix} = k \begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$, then k is equal to-

(A) 1 (B) 2 (C) 4 (D) 0

Sol. **[C]**

Proper Method

$$\begin{vmatrix} a^2 & b^2 & c^2 \\ (a+1)^2 & (b+1)^2 & (c+1)^2 \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix}$$

$R_2 \to R_2 - R_3$

$$= \begin{vmatrix} a^2 & b^2 & c^2 \\ 4a & 4b & 4c \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix}$$

$$= 4 \begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix}$$

$R_3 \to R_3 - (R_1 - 2R_2)$

$$= 4 \begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$$

$\therefore k = 4$

Short Trick

Put a = 0, b = 1, c = 2

$$\begin{vmatrix} 0 & 1 & 4 \\ 1 & 4 & 9 \\ 1 & 0 & 1 \end{vmatrix} = k \begin{vmatrix} 0 & 1 & 4 \\ 0 & 1 & 2 \\ 1 & 1 & 1 \end{vmatrix}$$

$\Rightarrow -(-8) + 4(-4) = k(-2)$

$\Rightarrow k = 4$

Q.7 If x is real number such that $\begin{vmatrix} x+1 & x+2 & x+\alpha \\ x+2 & x+3 & x+\beta \\ x+3 & x+4 & x+\gamma \end{vmatrix} = 0$ then α, β, γ, are in

(A) A.P. (B) G.P. (C) H.P. (D) None of these

Sol. **[A]**

Proper Method

Let, $R_2 = R_2 - \left(\frac{R_1 + R_3}{2}\right)$

$$\therefore \Delta \Rightarrow \begin{vmatrix} x+1 & x+2 & x+\alpha \\ 0 & 0 & \beta - \left(\frac{\alpha+\gamma}{2}\right) \\ x+3 & x+4 & x+\gamma \end{vmatrix} = 0$$

Short Trick

Put x = 0

$$\begin{vmatrix} 1 & 2 & \alpha \\ 2 & 3 & \beta \\ 3 & 4 & \gamma \end{vmatrix} = 0$$

$\Rightarrow 1(3\gamma - 4\beta) - 2(2\gamma - 3\beta) + \alpha(-1) = 0$

$\Rightarrow 2\beta = \alpha + \gamma$

Expanding along R_2, we get $\beta - \left(\frac{\alpha+\gamma}{2}\right) = 0$ $\therefore \alpha + \gamma = 2\beta$ $\therefore \alpha, \beta, \gamma$ are in AP	α, β, γ are in A.P.

Q.8 The value of the determinant $\begin{vmatrix} (x-2)^2 & (x-1)^2 & x^2 \\ (x-1)^2 & x^2 & (x+1)^2 \\ x^2 & (x+1)^2 & (x+2)^2 \end{vmatrix}$ is-

(A) 0 (B) $8x^2$ (C) 8 (D) –8

Sol. **[D]**

Proper Method

$$\begin{vmatrix} (x-2)^2 & (x-1)^2 & x^2 \\ (x-1)^2 & x^2 & (x+1)^2 \\ x^2 & (x+1)^2 & (x+2)^2 \end{vmatrix}$$

$C_1 \to C_1 - C_2$

$$= \begin{vmatrix} -2x+3 & (x-1)^2 & x^2 \\ -2x+1 & x^2 & (x+1)^2 \\ -2x-1 & (x+1)^2 & (x+2)^2 \end{vmatrix}$$

$C_2 \to C_2 - C_3$

$$= \begin{vmatrix} -2x+3 & -2x+1 & x^2 \\ -2x+1 & -2x-1 & (x+1)^2 \\ -2x-1 & -2x-3 & (x+2)^2 \end{vmatrix}$$

$C_1 \to C_1 - C_2$

$$\begin{vmatrix} 2 & -2x+1 & x^2 \\ 2 & -2x-1 & (x+1)^2 \\ 2 & -2x-3 & (x+2)^2 \end{vmatrix}$$

$R_1 \to R_1 - R_2,\ R_2 \to R_2 - R_3$

$$\begin{vmatrix} 0 & 2 & -2x-1 \\ 0 & 2 & -2x-3 \\ 2 & -2x-3 & (x+2)^2 \end{vmatrix}$$

$= 2\{-4x - 6 + 4x + 2\} = -8$

Short Trick

Put $x = 2$

$$\begin{vmatrix} 0 & 1 & 4 \\ 1 & 4 & 9 \\ 4 & 9 & 16 \end{vmatrix} = -1(-20) + 4(-7) = -8$$

Q.9 If $\begin{vmatrix} a+b & b+c & c+a \\ b+c & c+a & a+b \\ c+a & a+b & b+c \end{vmatrix} = \lambda \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ then λ is equal to -

(A) 1 (B) 2 (C) 3 (D) 4

Sol. **[B]**

Proper Method

$$\begin{vmatrix} a+b & b+c & c+a \\ b+c & c+a & a+b \\ c+a & a+b & b+c \end{vmatrix}$$

$C_1 \to C_1 - C_2,\ C_2 \to C_2 - C_3$

$$= \begin{vmatrix} a-c & b-a & c+a \\ b-a & c-b & a+b \\ c-b & a-c & b+c \end{vmatrix}$$

$C_1 \to C_1 + C_3$

$$= \begin{vmatrix} 2a & b-a & c+a \\ 2b & c-b & a+b \\ 2c & a-c & b+c \end{vmatrix}$$

$$= 2\begin{vmatrix} a & b-a & c+a \\ b & c-b & a+b \\ c & a-c & b+c \end{vmatrix}$$

$C_2 \to C_2 + C_1,\ C_3 \to C_3 - C_1$

$$= 2\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

$\therefore \lambda = 2$

Short Trick

Put $a = 0, b = 1, c = 2$

$$\Rightarrow \begin{vmatrix} 1 & 3 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{vmatrix} = \lambda \begin{vmatrix} 0 & 1 & 2 \\ 1 & 2 & 0 \\ 2 & 0 & 1 \end{vmatrix}$$

$\Rightarrow 1(5) - 3(7) + 2(-1)$

$= \lambda\{-1(1) + 2(-4)\}$

$\Rightarrow \lambda = 2$

Q.10 For any ΔABC, the value of determinant $\begin{vmatrix} \sin^2 A & \cot A & 1 \\ \sin^2 B & \cot B & 1 \\ \sin^2 C & \cot C & 1 \end{vmatrix}$ is-

(A) 0 (B) 1

(C) $\sin A \sin B \sin C$ (D) $\sin A + \sin B + \sin C$

Sol. **[A]**

Proper Method

$$\begin{vmatrix} \sin^2 A & \cot A & 1 \\ \sin^2 B & \cot B & 1 \\ \sin^2 C & \cot C & 1 \end{vmatrix}$$

$R_1 \to R_1 - R_2,\ R_2 \to R_2 - R_3$

$$\Rightarrow \begin{vmatrix} \sin^2 A - \sin^2 B & \cot A - \cot B & 0 \\ \sin^2 B - \sin^2 C & \cot B - \cot C & 0 \\ \sin^2 C & \cot C & 1 \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} \sin(A+B)\sin(A-B) & \dfrac{\sin(B-A)}{\sin A \cdot \sin B} & 0 \\ \sin(B+C)\sin(B-C) & \dfrac{\sin(C-B)}{\sin B \cdot \sin C} & 0 \\ \sin^2 C & \cot C & 1 \end{vmatrix}$$

Short Trick

Let $A = B = C = 60°$

$$\begin{vmatrix} 3/4 & 1/3 & 1 \\ 3/4 & 1/3 & 1 \\ 3/4 & 1/3 & 1 \end{vmatrix} = 0$$

$$= \sin(A-B)\sin(B-C)\begin{vmatrix} \sin(A+B) & \dfrac{-1}{\sin A\cdot\sin B} & 0 \\ \sin(B+C) & \dfrac{-1}{\sin B\cdot\sin C} & 0 \\ \sin^2 C & \cot C & 1 \end{vmatrix}$$

$$= \sin(A-B)\sin(B-C)\left\{\frac{-\sin(A+B)}{\sin B\cdot\sin C}+\frac{\sin(B+C)}{\sin A\cdot\sin B}\right\}$$

$$= \sin(A-B)\cdot\sin(B-C)\left\{\frac{-\sin C}{\sin B\cdot\sin C}+\frac{\sin A}{\sin A\cdot\sin B}\right\}$$

$$= \sin(A-B)\cdot\sin(B-C)\,(0) = 0$$

Q.11 If $S_r = \begin{vmatrix} 2r & x & n(n+1) \\ 6r^2-1 & y & n^2(2n+3) \\ 4r^3-2nr & z & n^3(n+1) \end{vmatrix}$ then $\sum_{r=1}^{n} S_r$ does not depends on

(A) x (B) y (C) n (D) all of these

Sol. **[D]**

Proper Method

$$\Sigma S_r = \begin{vmatrix} \Sigma 2r & x & n(n+1) \\ \Sigma(6r^2-1) & y & n^2(2n+3) \\ \Sigma(4r^3-2nr) & z & n^3(n+1) \end{vmatrix}$$

$\therefore \Sigma 2r = 2\Sigma r = n(n+1)$

& $\Sigma(6r^2-1) = 6\Sigma r^2 - \Sigma 1 = n(n+1)(2n+1) - n$

$= n[2n^2+3n+1-1] = n^2(2n+3)$

& $\Sigma(4r^3-2nr) = 4\Sigma r^3 - 2n\Sigma r$

$$= 4\left[\frac{n(n+1)}{2}\right]^2 - 2n\,\frac{n(n+1)}{2}$$

$= [n(n+1)]^2 - n^2(n+1)$

$= n^2(n+1)[(n+1)-1]$

$= n^3(n+1)$

$\because C_1$ & C_3 are identical so, $\Sigma S_r = 0$

Short Trick

Put r = 1, n = 1

$$S_1 = \begin{vmatrix} 2 & x & 2 \\ 5 & y & 5 \\ 2 & z & 2 \end{vmatrix} = 0$$

Q.12 In a ΔABC, if $\begin{vmatrix} 1 & a & b \\ 1 & c & a \\ 1 & b & c \end{vmatrix} = 0$ then $\sin^2 A + \sin^2 B + \sin^2 C$ is equal to-

(A) 9/4 (B) 4/9 (C) 1 (D) $3\sqrt{3}$

Sol. **[A]**

Proper Method

$$\begin{vmatrix} 1 & a & b \\ 1 & c & a \\ 1 & b & c \end{vmatrix} = 0$$

$\Rightarrow a^2+b^2+c^2-ab-bc-ca = 0$

$$\Rightarrow \frac{(a-b)^2+(b-c)^2+(c-a)^2}{2} = 0$$

$\therefore a = b = c$

$\therefore$ it is equilateral Δ so, $A = B = C = 60°$

$\therefore \sin^2 A + \sin^2 B + \sin^2 C = 9/4$

Short Trick

$$\begin{vmatrix} 1 & a & b \\ 1 & c & a \\ 1 & b & c \end{vmatrix} = 0 \text{ when } a = b = c$$

i.e. triangle is equilaterals

$\therefore \sin^2 A + \sin^2 B + \sin^2 C = \dfrac{9}{4}$

Q.13 If $a_1, a_2, a_3, \ldots\ldots, a_n, \ldots\ldots$ are in G.P., then the value of the determinant

$$\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$$, is- **[AIEEE-2004]**

(A) 0 (B) 1 (C) 2 (D) – 2

Sol. **[A]**

Proper Method

$\because a_1, a_2, a_3 \ldots\ldots a_n$ are in G.P.

$\therefore a_{n+1}^2 = a_n a_{n+2} \Rightarrow 2\log a_{n+1} = \log a_n + \log a_{n+2}$

$a_{n+4}^2 = a_{n+3} \cdot a_{n+5} \Rightarrow 2\log a_{n+4} = \log a_{n+3} + \log a_{n+5}$

$a_{n+7}^2 = a_{n+6} \cdot a_{n+8} \Rightarrow 2\log a_{n+7} = \log a_{n+6} + \log a_{n+8}$

$$= \begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$$

$C_1 \to C_1 + C_3$

$$= \begin{vmatrix} 2\log a_{n+1} & \log a_{n+1} & \log a_{n+2} \\ 2\log a_{n+4} & \log a_{n+4} & \log a_{n+5} \\ 2\log a_{n+7} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$$

$$= 2\begin{vmatrix} \log a_{n+1} & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+4} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+7} & \log a_{n+7} & \log a_{n+8} \end{vmatrix} = 0 \ (\because C_1 = C_2)$$

Short Trick

Let the G.P. be $2, 2^2, 2^3 2^4 \ldots$.

Let n = 1, then

$$\begin{vmatrix} \log 2 & \log 2^2 & \log 2^3 \\ \log 2^4 & \log 2^5 & \log 2^6 \\ \log 2^7 & \log 2^8 & \log 2^9 \end{vmatrix}$$

$$= (\log 2)^3 \begin{vmatrix} 2 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} = (\log 2)^3 \times 0 = 0$$

Q.14 If p^{th}, q^{th}, r^{th} term of a GP are ℓ, m, n then the value of $\begin{vmatrix} \log \ell & p & 1 \\ \log m & q & 1 \\ \log n & r & 1 \end{vmatrix}$ is equal to- **[AIEEE-2002]**

(A) 0 (B) 1 (C) $\ell + m + n$ (D) $p + q + r$

Sol. **[A]**

Proper Method

If first term = α, common ratio = β

$\ell = \alpha\beta^{p-1} \Rightarrow \log \ell = \log\alpha + (p-1)\log\beta$

$m = \alpha\beta^{q-1} \Rightarrow \log m = \log\alpha + (q-1)\log\beta$

$n = \alpha\beta^{r-1} \Rightarrow \log n = \log\alpha + (r-1)\log\beta$

$$\therefore \Delta = \begin{vmatrix} \log \ell & p & 1 \\ \log m & q & 1 \\ \log n & r & 1 \end{vmatrix}$$

Let $R_3 = R_3 - R_2$; $R_2 = R_2 - R_1$

$$\therefore \Delta = \begin{vmatrix} \log \ell & p & 1 \\ (q-p)\log\beta & q-p & 0 \\ (r-q)\log\beta & r-q & 0 \end{vmatrix}$$

$\therefore \Delta = 0$

Short Trick

Let p = 1, q = 2, r = 3

and $\ell = 2$, $m = 2^2$, $n = 2^3$

then $\begin{vmatrix} \log 2 & 1 & 1 \\ \log 2^2 & 2 & 1 \\ \log 2^2 & 3 & 1 \end{vmatrix} = \log 2 \begin{vmatrix} 1 & 1 & 1 \\ 2 & 2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = 0$

Q.15 If $a^2 + b^2 + c^2 = -2$ and $f(x) = \begin{vmatrix} 1+a^2x & (1+b^2)x & (1+c^2)x \\ (1+a^2)x & 1+b^2x & (1+c^2)x \\ (1+a^2)x & (1+b^2)x & 1+c^2x \end{vmatrix}$ then f(x) is a polynomial of degree –

[AIEEE-2005]

(A) 1 (B) 0 (C) 3 (D) 2

Sol. **[D]**

Proper Method

Let, $C_1 = C_1 + C_2 + C_3$

$$f(x) = \begin{vmatrix} 1 & (1+b^2)x & (1+c^2)x \\ 1 & 1+b^2x & (1+c^2)x \\ 1 & (1+b^2)x & 1+c^2x \end{vmatrix}$$

Let $R_3 = R_3 - R_2$; $R_2 = R_2 - R_1$

$$\therefore f(x) = \begin{vmatrix} 1 & (1+b^2)x & (1+c^2)x \\ 0 & 1-x & 0 \\ 0 & x-1 & 1-x \end{vmatrix}$$

$\therefore f(x) = (1-x)^2$

$\therefore$ Hence, Degree = 2

Short Trick

$a^2 + b^2 + c^2 = -2$

let $a^2 = 0$, $b^2 = -1$, $c^2 = -1$

$$\therefore f(x) = \begin{vmatrix} 1 & 0 & 0 \\ x & 1-x & 0 \\ x & 0 & 1-x \end{vmatrix}$$

$= 1 - 2x + x^2$

$\Rightarrow$ degree = 2

Q.16 Let a, b, c be such that $b(a + c) \neq 0$. If $\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^{n-2}a & (-1)^{n+1}b & (-1)^n c \end{vmatrix} = 0$,

then the value of n is :

[AIEEE-2009]

(A) any even integer (B) any odd integer (C) any integer (D) zero

Sol. **[B]**

Proper Method

$$\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + (-1)^n \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} = 0$$

n is odd integer then above relation is true

n is even integer then,

$$= \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} = 0$$

$$= \begin{vmatrix} a & 1 & -2 \\ -b & 2b+1 & -2 \\ c & -1 & 2 \end{vmatrix} = 0 \ (C_3 = C_3 - C_2,$$

$$= \begin{vmatrix} a+c & 0 & 0 \\ -b & 2b+1 & -2 \\ c & -1 & 2 \end{vmatrix} = 0 \ (R_1 \to R_1 + R_3)$$

$8b(a + c) = 0$ which is not true.

Short Trick

$b(a + c) \neq 0$

so let $a = b = c = 1$, then

$$\begin{vmatrix} 1 & 2 & 0 \\ -1 & 2 & 0 \\ 1 & 0 & 2 \end{vmatrix} + (-1)^n \begin{vmatrix} 2 & 2 & 0 \\ 0 & 0 & 2 \\ 1 & -1 & 1 \end{vmatrix} = 0$$

$\Rightarrow 1(4) - 2(-2) + (-1)^n \{2(2) - 2(-2)\} = 0$

$\Rightarrow 8 + (-1)^n 8 = 0$ is possible only if n is any odd integer

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Q.17 If $\begin{vmatrix} b^2+c^2 & ab & ac \\ ab & c^2+a^2 & bc \\ ac & bc & a^2+b^2 \end{vmatrix} = k\,a^2\,b^2\,c^2$, then k is equal to :

(A) 3 (B) 4 (C) 1 (D) 2

Sol. **[B]**

Proper Method

Let $\Delta = \begin{vmatrix} b^2+c^2 & ab & ac \\ ab & c^2+a^2 & bc \\ ac & bc & a^2+b^2 \end{vmatrix}$

Multiply C_1, C_2 and C_3 by a, b, c respectively

$$\Delta = \frac{1}{abc}\begin{vmatrix} a(b^2+c^2) & ab^2 & ac^2 \\ a^2b & b(c^2+a^2) & bc^2 \\ a^2c & cb^2 & c(a^2+b^2) \end{vmatrix}$$

$$= \frac{abc}{abc}\begin{vmatrix} b^2+c^2 & b^2 & c^2 \\ a^2 & c^2+a^2 & c^2 \\ a^2 & b^2 & a^2+b^2 \end{vmatrix}$$

Apply $C_1 \to C_1 - C_2 - C_3$

$$\Delta = \begin{vmatrix} 0 & b^2 & c^2 \\ -2c^2 & c^2+a^2 & c^2 \\ -2b^2 & b^2 & a^2+b^2 \end{vmatrix} = 4a^2b^2c^2$$

$\therefore k = 4$

Short Trick

Put a = b = c = 1,

both sides then

$$\begin{vmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix} = k$$

$\Rightarrow k = 2(4-1) - 1(1) + (-1) = 4$

Q.18 If $D_r = \begin{vmatrix} 2^{r-1} & 2.3^{r-1} & 4.5^{r-1} \\ \alpha & \beta & \gamma \\ 2^n-1 & 3^n-1 & 5^n-1 \end{vmatrix}$, then the value of $\sum_{r=1}^{n} D_r$ is- **[IIT-1993]**

(A) 0 (B) $\alpha\beta\gamma$ (C) $\alpha+\beta+\gamma$ (D) $\alpha.\,2^n + \beta.\,3^n + \gamma.\,4^n$

Sol. **[A]**

Proper Method

$$\Sigma D_r = \begin{vmatrix} \Sigma 2^{r-1} & 2\Sigma 3^{r-1} & 4\Sigma 5^{r-1} \\ \alpha & \beta & \gamma \\ 2^n-1 & 3^n-1 & 5^n-1 \end{vmatrix}$$

clearly $\Sigma 2^{r-1} = 1.\ \frac{(2^n-1)}{2-1} = 2^n-1$

& $2.\ \Sigma 3^{r-1} = 2.\ \frac{(3^n-1)}{3-1} = 3^n-1$

& $4.\Sigma 5^{r-1} = 4.\ \frac{(5^n-1)}{5-1} = 5^n-1$

$\therefore R_1$ & R_3 are identical so, $\Sigma D_r = 0$

Short Trick

Let r = 1, n = 1

$$D_1 = \begin{vmatrix} 1 & 2 & 4 \\ \alpha & \beta & \gamma \\ 1 & 2 & 4 \end{vmatrix} = 0$$

Q.19 If $\begin{vmatrix} -2a & a+b & a+c \\ b+a & -2b & b+c \\ c+a & b+c & -2c \end{vmatrix} = \alpha(a+b)(b+c)(c+a) \neq 0$, then α is equal to **[AIEEE Online-2012]**

(A) 1 (B) a + b + c (C) abc (D) 4

Sol. **[D]**

Proper Method

Let $\Delta = \begin{vmatrix} -2a & a+b & a+c \\ b+a & -2b & b+c \\ c+a & b+c & -2c \end{vmatrix}$

Applying $C_1 + C_3$ and $C_2 + C_3$

$\Delta = \begin{vmatrix} -a+c & 2a+b+c & a+c \\ 2b+a+c & -b+c & b+c \\ a-c & b-c & -2c \end{vmatrix}$

Now, applying $R_1 + R_3$ and $R_2 + R_3$

$\Delta = \begin{vmatrix} 0 & 2(a+b) & a-c \\ 2(a+b) & 0 & b-c \\ a-c & b-c & -2c \end{vmatrix}$

On expanding, we get

$\Delta = -2(a+b)\{-2c[2(a+b)] - (a-c)(b-c)\}$
$+ (a-c)[2(a+b)(b-c)]$

$\Delta = 8c(a+b)(a+b) + 4(a+b)(a-c)(b-c)$

$= 4(a+b)[2ac + 2bc + ab - bc - ac + c^2]$

$= 4(a+b)[ac + bc + ab + c^2]$

$= 4(a+b)[c(a+c) + b(a+c)]$

$= 4(a+b)(b+c)(c+a)$

$= \alpha(a+b)(b+c)(c+a)$

Hence, $\alpha = 4$

Short Trick

Let a = 0, b = 1, c = 2

$\begin{vmatrix} 0 & 1 & 2 \\ 1 & -2 & 3 \\ 2 & 3 & -4 \end{vmatrix} = \alpha(1)(3)(2)$

$\Rightarrow -1(-10) + 2(7) = 6\alpha$

$\Rightarrow \alpha = 4$

Q.20 If a, b, c are sides of scalene triangle, then the value of $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is : **[JEE Main Online-2013]**

(A) non-negative (B) negative (C) positive (D) non-positive

Sol. **[B]**

Proper Method

$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ b & c & a \\ c & a & b \end{vmatrix}$

$= (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ b & c & a \\ c & a & b \end{vmatrix}$

$= (a+b+c)\begin{vmatrix} 0 & 0 & 1 \\ b-c & c-a & a \\ c-a & a-b & b \end{vmatrix}$

Short Trick

$\because$ a, b, c are sides of scalene triangle

So let a = 1, b = 2, c = 3

$\begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{vmatrix} = -18$ (negative)

$= (a+b+c)\,[ab+bc+ca-a^2-b^2-c^2]$
$= -(a+b+c)\,[(a-b)^2+(b-c)^2+(c-a)^2]$
Since a, b, c are sides of a scalene triangle, therefore at least two of the a, b, c will be unequal.
$\therefore (a-b)^2+(b-c)^2+(c-a)^2 > 0$
Also $a+b+c=0$
$\therefore -(a+b+c)\,[(a-b)^2+(b-c)^2+(c-a)^2] < 0$

Q.21 If $\Delta_r = \begin{vmatrix} r & 2r-1 & 3r-2 \\ \frac{n}{2} & n-1 & a \\ \frac{1}{2}n(n-1) & (n-1)^2 & \frac{1}{2}(n-1)(3n+4) \end{vmatrix}$, then the value of $\sum_{r=1}^{n-1}\Delta_r$: **[JEE Main Online-2014]**

(A) Depends only on a
(B) Depends only on n
(C) Depends both on a and n
(D) Is independent of both a and n

Sol. **[D]**

Proper Method

$\sum_{r=1}^{n-1} r = 1+2+3+\ldots+(n-1) = \frac{n(n-1)}{2}$

$\sum_{r=1}^{n-1}(2r-1) = 1+3+5+\ldots+[2(n-1)-2] = (n-1)^2$

$\sum_{r=1}^{n-1}(3r-2) = 1+4+7+\ldots+(3n-3-2)$

$= \frac{(n-1)(3n-4)}{2}$

$\therefore \sum_{r=1}^{n-1}\Delta_r = \begin{vmatrix} \Sigma r & \Sigma(2r-1) & \Sigma(3r-2) \\ \frac{n}{2} & n-1 & a \\ \frac{n(n-1)}{2} & (n-1)^2 & \frac{(n-1)(3n-4)}{2} \end{vmatrix}$

$\sum_{r=1}^{n-1}\Delta_r$ consists of $(n-1)$ determinants in L.H.S. and in R.H.S. every constituent of first row consist of $(n-1)$ elements and hence it can be splitted into sum of $(n-1)$ determinants.

$\therefore \sum_{r=1}^{n-1}\Delta_r = \begin{vmatrix} \frac{n(n-1)}{2} & (n-1)^2 & \frac{(n-1)(3n-4)}{2} \\ \frac{n}{2} & n-1 & a \\ \frac{n(n-1)}{2} & (n-1)^2 & \frac{(n-1)(3n-4)}{2} \end{vmatrix}$

$= 0$

($\because$ R_1 and R_3 are identical)

Hence, value of $\sum_{r=1}^{n-1}\Delta_r$ is independent of both 'a' and 'n'.

Short Trick

$r = 1,\ n = 2$

$\Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & a \\ 1 & 1 & 5 \end{vmatrix} = 0$

Q.22 If a, b, c are in G.P., then the value of determinant $\Delta = \begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ ax+b & bx+c & 0 \end{vmatrix}$ is- **[IIT-1993]**

(A) 1 (B) 0 (C) -1 (D) -2

Sol. **[B]**

Proper Method

Let $R_3 = R_3 - (xR_1 + R_2)$

$$\therefore \Delta = \begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ 0 & 0 & -[x(ax+b)+(bx+c)] \end{vmatrix}$$

expanding along R_3

$\therefore = -[x(ax+b)+(bx+c)](ac-b^2)$

but a,b,c are in GP so, $\Delta = 0$

Short Trick

Let a = 1, b = 2, c = 4 and x = 0

$$\Delta = \begin{vmatrix} 1 & 2 & 2 \\ 2 & 4 & 4 \\ 2 & 4 & 0 \end{vmatrix}$$

$\Delta = 1(-16) - 2(-8) + 2(0) = 0$

Q.23 If $f(x) = \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & (x+1)x \\ 3x(x-1) & x(x-1)(x-2) & (x+1)x(x-1) \end{vmatrix}$ then f(100) is equal to - **[IIT-1999]**

(A) 0 (B) 1 (C) 100 (D) -100

Sol. **[A]**

Proper Method

$$f(x)\begin{vmatrix} 1 & x & (x+1) \\ 2x & x(x-1) & (x+1)x \\ 3x(x-1) & x(x-1)(x-2) & (x+1)x(x-1) \end{vmatrix}\begin{matrix} \\ \to x \\ \to x(x-1) \end{matrix}$$

$$f(x) = x^2(x-1)\begin{vmatrix} 1 & x & (x+1) \\ 2 & (x-1) & (x+1) \\ 3 & (x-2) & (x+1) \end{vmatrix}$$

Let $R_3 = R_3 - R_2$, $R_2 = R_2 - R_1$

$$f(x) = x^2(x-1)\begin{vmatrix} 1 & x & x+1 \\ 1 & -1 & 0 \\ 1 & -1 & 0 \end{vmatrix} = 0$$

$\therefore f(100) = 0$

Short Trick

$\because$ here f(0) = f(1) = f(–1) = ... = 0

$\therefore$ The value of f(x) is independent of x so, it is an identity in x, so put x = 0

$$f(0) = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{vmatrix} = 0$$

$\Rightarrow f(x) = 0$

$\Rightarrow f(100) = 0$

Q.24 If a, b, c, d, e and f are in G.P. then the value of $\begin{vmatrix} a^2 & d^2 & x \\ b^2 & e^2 & y \\ c^2 & f^2 & z \end{vmatrix}$ depends on

(A) x and y (B) x and z (C) y and z (D) Independent of x, y & z

Sol. **[D]**

Proper Method

$b = ar$, $c = ar^2$, $d = ar^3$, $e = ar^4$, $f = ar^5$

$$\text{Determinant} = \begin{vmatrix} a^2 & a^2r^6 & x \\ a^2r^2 & a^2r^8 & y \\ a^2r^4 & a^2r^{10} & z \end{vmatrix}$$

$$= a^2a^2r^6\begin{vmatrix} 1 & 1 & x \\ r^2 & r^2 & y \\ r^4 & r^4 & z \end{vmatrix} = 0$$

Short Trick

Let a = 1, b = 2, c = 2^2, d = 2^3, e = 2^4, f = 2^5

$$= \begin{vmatrix} 1 & 2^6 & x \\ 2^2 & 2^8 & y \\ 2^4 & 2^{10} & z \end{vmatrix}$$

$$= 2^6\begin{vmatrix} 1 & 1 & x \\ 2^2 & 2^2 & y \\ 2^4 & 2^4 & z \end{vmatrix} = 0$$

Q.25 $\begin{vmatrix} a & a+b & a+2b \\ a+2b & a & a+b \\ a+b & a+2b & a \end{vmatrix} = mb^n (a+b)$, then

(A) m = 9, n =1 (B) m = 1, n = 2 (C) m = 9, n = 2 (D) m = 9, n = 3

Sol. **[C]**

Proper Method

$$\Delta = \begin{vmatrix} a & a+b & a+2b \\ a+2b & a & a+b \\ a+b & a+2b & a \end{vmatrix}$$

$$\Delta = 3(a+b)\begin{vmatrix} 1 & a+b & a+2b \\ 1 & a & a+b \\ 1 & a+2b & a \end{vmatrix} C_1 \to C_1 + C_2 + C_3$$

$$= 3(a+b)\begin{vmatrix} 1 & a+b & a+2b \\ 0 & -b & -b \\ 0 & b & -2b \end{vmatrix}$$

$= 9b^2 (a+b) = mb^n (a+b)$

$\therefore$ m = 9, n = 2

Short Trick

Let a = 1, b = 2

$$\begin{vmatrix} 1 & 3 & 5 \\ 5 & 1 & 3 \\ 3 & 5 & 1 \end{vmatrix} = m.2^n(3)$$

$\Rightarrow 1(-14) - 3(-4) + 5(22) = m.2^n(3)$

$\Rightarrow 108 = m.2^n(3)$

$\Rightarrow 9 \times 2^2 (3) = m.2^n(3)$

m= 9, n = 2

Q.26 Let $\Delta_r = \begin{vmatrix} 2^{r-1} & \dfrac{(r+1)!}{1+\dfrac{1}{r}} & 2r \\ a & b & c \\ 2^n - 1 & (n+1)!-1 & n(n+1) \end{vmatrix}$, then value of $\sum_{r=1}^{n} \Delta_r$ is

(A) 0 (B) (n + 3)! (C) a (n)! + b (D) (n + 2)!

Sol. **[A]**

Proper Method

$$\Delta_r = \begin{vmatrix} 2^{r-1} & r(r!) & 2r \\ a & b & c \\ 2^n - 1 & (n+1)!-1 & n(n+1) \end{vmatrix}$$

$$\sum_{r=1}^{n} \Delta_r = \begin{vmatrix} \sum_{r=1}^{n} 2^{r-1} & \sum_{r=1}^{n} r(r!) & \sum_{r=1}^{n} 2r \\ a & b & c \\ 2^n - 1 & (n+1)!-1 & n(n+1) \end{vmatrix} = 0$$

$\because \sum_{r=1}^{n} 2^{r-1} = 2^n - 1$

$\sum_{r=1}^{n} r(r!) = (n+1)! - 1$

Short Trick

r = 1, n = 1

$$\Delta_1 = \begin{vmatrix} 1 & 1 & 2 \\ a & b & c \\ 1 & 1 & 2 \end{vmatrix} = 0$$

Q.27 Let $\Delta = \begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & a+b & c \end{vmatrix}$. Then $\Delta =$

(A) $(a+b+c)(a^2+b^2+c^2)$ (B) $a^3+b^3+c^3$

(C) $a^2+b^2+c^2$ (D) 0

Sol. **[A]**

Proper Method

$$\Delta = \begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & a+b & c \end{vmatrix}$$

$C_1 \to aC_1, C_2 \to bC_2, C_3 \to cC_3$ and then $C_1 \to C_1 + C_2 + C_3$, and taking $(a^2+b^2+c^2)$ common, we get

$$\Delta = \frac{(a^2+b^2+c^2)}{abc}\begin{vmatrix} 1 & b^2-bc & c^2+bc \\ 1 & b^2 & c^2-ac \\ 1 & b^2+ab & c^2 \end{vmatrix}$$

$$= \frac{a^2+b^2+c^2}{a}\begin{vmatrix} 1 & b-c & c+b \\ 1 & b & c-a \\ 1 & b+a & c \end{vmatrix}$$

$$= \frac{a^2+b^2+c^2}{a}\begin{vmatrix} 1 & b-c & c+b \\ 0 & c & -a-b \\ 0 & a+c & -b \end{vmatrix}$$

$[R_2 \to R_2 - R_1, R_3 \to R_3 - R_1]$

$$= \frac{a^2+b^2+c^2}{a}\{-bc + a^2 + ac + ab + bc\}$$

$= (a+b+c)(a^2+b^2+c^2)$

Short Trick

Let $a = 0, b = 1, c = 2$

$$\begin{vmatrix} 0 & -1 & 3 \\ 2 & 1 & 2 \\ -1 & 1 & 2 \end{vmatrix}$$

$= +1(6) + 3(3) = 15$

by option (A) $(a+b+c)(a^2+b^2+c^2)$ is correct option

Q.28 The value of the determinant $\begin{vmatrix} x & x+a & x+2a \\ x+1 & x+2a & x+4a \\ x+2 & x+3a & x+6a \end{vmatrix}$ is :

(A) 0 (B) $a^3 - x^3$ (C) $x^3 - a^3$ (D) $(x-a)^3$

Sol. **[A]**

Proper Method

$(R_2 \to R_2 - R_1; R_3 \to R_3 - R_2)$

$$\Delta = \begin{vmatrix} x & x+a & x+2a \\ 1 & a & 2a \\ 1 & a & 2a \end{vmatrix}$$

$= 0$

Short Trick

Let $a = 1, x = 2$

$$\begin{vmatrix} 2 & 3 & 4 \\ 3 & 4 & 6 \\ 4 & 5 & 8 \end{vmatrix} = 2(2) - 3(0) + 4(-1) = 0$$

Q.29 If $\begin{vmatrix} x^n & x^{n+2} & x^{n+3} \\ y^n & y^{n+2} & y^{n+3} \\ z^n & z^{n+2} & z^{n+3} \end{vmatrix} = (y-z)(z-x)(x-y)\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$, then :

(A) $n = 2$ (B) $n = -2$ (C) $n = -1$ (D) $n = 1$

Sol. **[C]**

Proper Method

$$\Delta = x^n y^n z^n \begin{vmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{vmatrix}$$

$$= x^n y^n z^n \begin{vmatrix} 1 & x^2 & x^3 \\ 0 & y^2 - x^2 & y^3 - x^3 \\ 0 & z^2 - x^2 & z^3 - x^3 \end{vmatrix}$$

$$= x^n y^n z^n (y - x)(z - x) \begin{vmatrix} 1 & x^2 & x^3 \\ 0 & y + x & y^2 + x^2 + yx \\ 0 & z + x & z^2 + x^2 + zx \end{vmatrix}$$

$$= x^n y^n z^n (y - x)(z - x) \begin{vmatrix} 1 & x^2 & x^3 \\ 0 & y + x & y^2 + x^2 + yx \\ 0 & z - y & z^2 - y^2 + x(z - y) \end{vmatrix}$$

$$= x^n y^n z^n (z - x)(y - x)(z - y) \begin{vmatrix} 1 & x^2 & x^3 \\ 0 & y + x & y^2 + x^2 + yx \\ 0 & 1 & z + y + x \end{vmatrix}$$

$$= x^n y^n z^n (x - y)(y - z)(z - x)(xy + yz + zx)$$

$$= x^n y^n z^n (xyz)(x - y)(y - z)(z - x)\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right)$$

$$= (x - y)(y - z)(z - x)\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right)$$

$\Rightarrow n + 1 = 0 \Rightarrow n = -1$

Short Trick

Sum of degree of element of principal diagonal = degree of function on R.H.S

$\Rightarrow n + (n + 2) + (n + 3) = (1 + 1 + 1 - 1)$

$\Rightarrow n = -1$

Trick -2 Combination of method of substitution and balancing

Q.30 The value of $\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix}$ is equal to -

(A) a^3 (B) b^3 (C) c^3 (D) $a^3 + b^3 + c^3$

Sol. **[A]**

Proper Method

$$\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix}$$

$R_2 \to R_2 - 2R_1, R_3 \to R_3 - 3R_1$

$$= \begin{vmatrix} a & a+b & a+b+c \\ 0 & a & 2a+b \\ 0 & 3a & 7a+3b \end{vmatrix}$$

$= a(7a^2 + 3ab - 6a^2 - 3ab)$

$= a(a^2)$

$= a^3$

Short Trick

Put a = 0, b = 1, c = 2

then value of determinant = 0 then option (A) a^3 is correct answer

Q.31 The value of the determinant $\begin{vmatrix} ka & k^2+a^2 & 1 \\ kb & k^2+b^2 & 1 \\ kc & k^2+c^2 & 1 \end{vmatrix}$ is -

(A) $k(a+b)(b+c)(c+a)$ (B) $k\,abc\,(a^2+b^2+c^2)$

(C) $k(a-b)(b-c)(c-a)$ (D) $k(a+b-c)(b+c-a)(c+a-b)$

Sol. **[C]**

Proper Method

$$\Delta = \begin{vmatrix} ka & k^2+a^2 & 1 \\ kb & k^2+b^2 & 1 \\ kc & k^2+c^2 & 1 \end{vmatrix}$$

$$\therefore \Delta = \begin{vmatrix} ka & k^2 & 1 \\ kb & k^2 & 1 \\ kc & k^2 & 1 \end{vmatrix} + \begin{vmatrix} ka & a^2 & 1 \\ kb & b^2 & 1 \\ kc & c^2 & 1 \end{vmatrix}$$

$$\Delta = k^2 \begin{vmatrix} ka & 1 & 1 \\ kb & 1 & 1 \\ kc & 1 & 1 \end{vmatrix} + k \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix}$$

$\Delta = 0 + k(a-b)(b-c)(c-a)$

Short Trick

Put $a = 0, b = 1, c = 2$

$$\begin{vmatrix} 0 & k^2 & 1 \\ k & k^2+1 & 1 \\ 2k & k^2+4 & 1 \end{vmatrix}$$

$= -k^2(-k) + 1(k^3 + 4k - 2k^3 - 2k) = 2k$

by option (C) $k(a-b)(b-c)(c-a)$ is correct option

Q.32 The value of the determinant $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ bc & ca & ab \end{vmatrix}$ is -

(A) $abc(a-b)(b-c)(c-a)$ (B) $(a-b)(b-c)(c-a)(a+b+c)$

(C) $(a-b)(b-c)(c-a)(ab+bc+ca)$ (D) 0

Sol. **[C]**

Proper Method

$$\Delta = \begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ bc & ca & ab \end{vmatrix}$$

$$= \frac{abc}{abc} \begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ bc & ca & ab \end{vmatrix}$$

$$\Delta = \frac{1}{abc} \begin{vmatrix} a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \\ abc & abc & abc \end{vmatrix}$$

$$\therefore \Delta = \begin{vmatrix} a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \\ 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

$\Delta = (a-b)(b-c)(c-a)(ab+bc+ca)$

Short Trick

Put $a = 0, b = 1, c = 2$

$$\begin{vmatrix} 0 & 1 & 2 \\ 0 & 1 & 4 \\ 2 & 0 & 0 \end{vmatrix} = 4$$

by option (C) $(a-b)(b-c)(c-a)(ab+bc+ca)$ is correct option

Q.33 The value of $\begin{vmatrix} a & b+c & a^3 \\ b & c+a & b^3 \\ c & a+b & c^3 \end{vmatrix}$ is-

(A) $(a-b)(b-c)(c-a)$ (B) $abc(a-b)(b-c)(c-a)$

(C) $-(a+b+c)^2(a-b)(b-c)(c-a)$ (D) 0

Sol. **[C]**

Proper Method

$$\Delta = \begin{vmatrix} a & b+c & a^3 \\ b & c+a & b^3 \\ c & a+b & c^3 \end{vmatrix}$$

Let $C_2 = C_1 + C_2$

$$\therefore \Delta = \begin{vmatrix} a & a+b+c & a^3 \\ b & a+b+c & b^3 \\ c & a+b+c & c^3 \end{vmatrix}$$

$$\therefore \Delta = (a+b+c)\begin{vmatrix} a & 1 & a^3 \\ b & 1 & b^3 \\ c & 1 & c^3 \end{vmatrix}$$

$$\therefore \Delta = -(a+b+c)\begin{vmatrix} 1 & a & a^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix}$$

$\Delta = -(a+b+c)[(a-b)(b-c)(c-a)(a+b+c)]$

Short Trick

$a = 0, b = 1, c = 2$

$$\begin{vmatrix} 0 & 3 & 0 \\ 1 & 2 & 1 \\ 2 & 1 & 8 \end{vmatrix} = -18$$

by option (c) $-(a+b+c)^2(a-b)(b-c)(c-a)$ is correct option

Q.34 $\begin{vmatrix} \frac{a^2+b^2}{c} & c & c \\ a & \frac{b^2+c^2}{a} & a \\ b & b & \frac{c^2+a^2}{b} \end{vmatrix}$ is equal to-

(A) abc (B) 2abc (C) 4abc (D) 0

Sol. **[C]**

Proper Method

$$\begin{vmatrix} \frac{a^2+b^2}{c} & c & c \\ a & \frac{b^2+c^2}{a} & a \\ b & b & \frac{c^2+a^2}{b} \end{vmatrix}$$

$$= \frac{1}{abc}\begin{vmatrix} a^2+b^2 & c^2 & c^2 \\ a^2 & b^2+c^2 & a^2 \\ b^2 & b^2 & c^2+a^2 \end{vmatrix}$$

$R_1 \to R_1 - (R_2 + R_3)$

$$= \frac{1}{abc}\begin{vmatrix} 0 & -2b^2 & -2a^2 \\ a^2 & b^2+c^2 & a^2 \\ b^2 & b^2 & c^2+a^2 \end{vmatrix}$$

Short Trick

Put $a = -1, b = 1, c = 2$

$$\begin{vmatrix} 1 & 2 & 2 \\ -1 & -5 & -1 \\ 1 & 1 & 5 \end{vmatrix}$$

$= 1(-24) - 2(-4) + 2(4)$

$= -8$

by option (C) 4abc is correct option

$$= \frac{-2}{abc}\begin{vmatrix} 0 & b^2 & a^2 \\ a^2 & b^2+c^2 & a^2 \\ b^2 & b^2 & c^2+a^2 \end{vmatrix}$$

$R_2 \to R_2 - R_1$

$$= \frac{-2}{abc}\begin{vmatrix} 0 & b^2 & a^2 \\ a^2 & c^2 & 0 \\ b^2 & b^2 & c^2+a^2 \end{vmatrix}$$

$R_3 \to R_3 - R_1$

$$= \frac{-2}{abc}\begin{vmatrix} 0 & b^2 & a^2 \\ a^2 & c^2 & 0 \\ b^2 & 0 & c^2 \end{vmatrix}$$

$$= -\frac{-2}{abc}[0 - b^2(a^2c^2) + a^2(-b^2c^2)]$$

$$= \frac{4a^2b^2c^2}{abc}$$

$$= 4abc$$

Q.35 If a, b, c are non-zero real numbers, then $\begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{vmatrix}$ is equal to

(A) abc (B) $a^2b^2c^2$ (C) ab + bc + ca (D) 0

Sol. [D]

Proper Method

$$\begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{vmatrix}$$

$R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$

$$= \begin{vmatrix} c^2(b^2-a^2) & c(b-a) & (b-a) \\ a^2(c^2-b^2) & a(c-b) & (c-b) \\ a^2b^2 & ab & a+b \end{vmatrix}$$

$$= (b-a)(c-b)\begin{vmatrix} c(b+a) & c & 1 \\ a(c+b) & a & 1 \\ a^2b^2 & ab & a+b \end{vmatrix}$$

$= R_1 \to R_1 - R_2$

$$(b-a)(c-b)\begin{vmatrix} b(c-a) & c-a & 0 \\ a(c+b) & a & 1 \\ a^2b^2 & ab & a+b \end{vmatrix}$$

$$= (b-a)(c-b)(c-a)\begin{vmatrix} b & 1 & 0 \\ a(c+b) & a & 1 \\ a^2b^2 & ab & a+b \end{vmatrix}$$

$$= (b-a)(c-b)(c-a)\{b(a^2+ab-ab) - 1(a^2c+a^2b+abc+ab^2-a^2b^2)\}$$

$= 0$

Short Trick

Put a = 1, b = – 1, c = 2

$$\begin{vmatrix} 4 & -2 & 1 \\ 4 & 2 & 3 \\ 1 & -1 & 0 \end{vmatrix}$$

= 4(3) + 2(– 3) + 1 (– 6) = 0

by option (D) is correct option

Q.36 $\begin{vmatrix} x & p & q \\ p & x & q \\ p & q & x \end{vmatrix}$ is equal to -

(A) $(x + p)(x + q)(x - p - q)$ (B) $(x - p)(x - q)(x + p + q)$

(C) $(x - p)(x - q)(x - p - q)$ (D) $(x + p)(x + q)(x + p + q)$

Sol. **[B]**

Proper Method

$$\Delta = \begin{vmatrix} x & p & q \\ p & x & q \\ p & q & x \end{vmatrix}$$

Let $C_1 = C_1 + C_2 + C_3$

$$\Delta = \begin{vmatrix} x+p+q & p & q \\ x+p+q & x & q \\ x+p+q & q & x \end{vmatrix}$$

Let $R_3 = R_3 - R_2$; $R_2 = R_2 - R_1$

$$\Delta = \begin{vmatrix} x+p+q & p & q \\ 0 & x-p & 0 \\ 0 & q-x & x-q \end{vmatrix}$$

$$\therefore \Delta = (x + p + q)(x - p)(x - q) \begin{vmatrix} 1 & p & q \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{vmatrix}$$

$\therefore \Delta = (x - p)(x - q)(x + p + q)$

Short Trick

Put $x = 0$, $p = 1$, $q = 2$

$$\begin{vmatrix} 0 & 1 & 2 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{vmatrix} = -1(-2) + 2(2) = 6$$

by option (B) $(x - p)(x - q)(x + p + q)$ is correct option

Q.37 If $\begin{vmatrix} a^2 & b^2 & c^2 \\ (a+\lambda)^2 & (b+\lambda)^2 & (c+\lambda)^2 \\ (a-\lambda)^2 & (b-\lambda)^2 & (c-\lambda)^2 \end{vmatrix} = k\lambda \begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$, $\lambda \neq 0$. then k is equal to : **[JEE Main Online-2014]**

(A) $4\lambda abc$ (B) $-4\lambda abc$ (C) $4\lambda^2$ (D) $-4\lambda^2$

Sol. **[C]**

Proper Method

$$\text{Let } \Delta = \begin{vmatrix} a^2 & b^2 & c^2 \\ (a+\lambda)^2 & (b+\lambda)^2 & (c+\lambda)^2 \\ (a-\lambda)^2 & (b-\lambda)^2 & (c-\lambda)^2 \end{vmatrix}$$

Apply $R_2 \to R_2 - R_3$

$$\Delta = \begin{vmatrix} a^2 & b^2 & c^2 \\ (a+\lambda)^2 - (a-\lambda)^2 & (b+\lambda)^2 - (b-\lambda)^2 & (c+\lambda)^2 - (c-\lambda)^2 \\ (a-\lambda)^2 & (b-\lambda)^2 & (c-\lambda)^2 \end{vmatrix}$$

$$\Delta = \begin{vmatrix} a^2 & b^2 & c^2 \\ 4a\lambda & 4b\lambda & 4c\lambda \\ (a-\lambda)^2 & (b-\lambda)^2 & (c-\lambda)^2 \end{vmatrix}$$

($\because (x + y)^2 - (x - y)^2 = 4xy$)

Taking out 4 common from R_2

Short Trick

Let $a = 1$, $b = 2$, $c = 3$ and $\lambda = 1$

$$\therefore \begin{vmatrix} 1 & 4 & 9 \\ 4 & 9 & 16 \\ 0 & 1 & 4 \end{vmatrix} = k \begin{vmatrix} 1 & 4 & 9 \\ 1 & 2 & 3 \\ 1 & 1 & 1 \end{vmatrix}$$

$\Rightarrow -8 = -2k$

$\Rightarrow k = 4$

Put these value in options then (C) $4\lambda^2$ is correct option

$$= 4\begin{vmatrix} a^2 & b^2 & c^2 \\ a\lambda & b\lambda & c\lambda \\ a^2+\lambda^2-2a\lambda & b^2+\lambda^2-2b\lambda & c^2+\lambda^2-2c\lambda \end{vmatrix}$$

Apply $R_3 \to [R_3 - (R_1 - 2R_2)]$

$$= 4\begin{vmatrix} a^2 & b^2 & c^2 \\ a\lambda & b\lambda & c\lambda \\ \lambda^2 & \lambda^2 & \lambda^2 \end{vmatrix}$$

Taking out λ common from R_2 and λ^2 from R_3.

$$= 4\lambda(\lambda^2)\begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$$

$$= k\lambda\begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$$

$\Rightarrow k = 4\lambda^2$

Q.38 If $\alpha, \beta \neq 0$, and $f(n) = \alpha^n + \beta^n$ and $\begin{vmatrix} 3 & 1+f(1) & 1+f(2) \\ 1+f(1) & 1+f(2) & 1+f(3) \\ 1+f(2) & 1+f(3) & 1+f(4) \end{vmatrix} = K(1-\alpha)^2(1-\beta)^2(\alpha-\beta)^2$, then K is equal to - **[JEE Main-2014]**

(A) -1 (B) $\alpha\beta$ (C) $\frac{1}{\alpha\beta}$ (D) 1

Sol. **[D]**

Proper Method

$f(n) = \alpha^n + \beta^n$

$$\begin{vmatrix} 3 & 1+\alpha+\beta & 1+\alpha^2+\beta^2 \\ 1+\alpha+\beta & 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 \\ 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 & 1+\alpha^4+\beta^4 \end{vmatrix}$$

$= K(1-\alpha)^2(1-\beta)^2(\alpha-\beta)^2$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix} \cdot \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \beta & \beta^2 \end{vmatrix}$$

$= K(1-\alpha)^2(1-\beta)^2(\alpha-\beta)^2$

$$\begin{vmatrix} 0 & 0 & 1 \\ 1-\alpha & \alpha-\beta & \beta \\ 1-\alpha^2 & \alpha^2-\beta^2 & \beta^2 \end{vmatrix}^2$$

$= K(1-\alpha)^2(1-\beta)^2(\alpha-\beta)^2$

$$(1-\alpha)^2(\alpha-\beta)^2\begin{vmatrix} 1 & 1 \\ 1+\alpha & \alpha+\beta \end{vmatrix}^2$$

$= K(1-\alpha)^2(1-\beta)^2(\alpha-\beta)^2$

$(\beta-1)^2 = K(1-\beta)^2 \quad \Rightarrow K = 1$

Short Trick

Let $\alpha = 2, \beta = -1$, then $f(n) = 2^n + (-1)^n$

$f(1) = 2^1 + (-1)^1 = 1,$

$f(2) = 2^2 + (-1)^2 = 5,$

$f(3) = 2^3 + (-1)^3 = 7,$

$f(4) = 2^4 + (-1)^4 = 17$

$$\begin{vmatrix} 3 & 2 & 6 \\ 2 & 6 & 8 \\ 6 & 8 & 18 \end{vmatrix} = k(1-2)^2(1+1)^2(2+1)^2 = 36k$$

$\Rightarrow 36 = 36k$

$\Rightarrow k = 1$